of the potential wall. To show this let n
ion (3), giving:

$$-\left\{\frac{1}{4}+\frac{l(l+1)}{\rho^2}\right\}R=0 \qquad (54)$$

$= 2rn^{-1} = 2ir\sqrt{2E}$ (2)

$$+\left\{2E-\frac{l(l+1)}{r^2}\right\}R=0. \qquad (55)$$

tion of the particle in a spherical box.
re the Bessel-functions

$$_{l+\frac{1}{2}}(r\sqrt{2E}), \qquad (56)$$

d as an asymptotic case of (4) for $E\rightarrow\infty$
s.

function (56) has nodes at $r_0\sqrt{2E}=q\pi$
or the energy curve of the $1s$ level the

$$E=\pi^2/2r_0^2 \qquad (57)$$

te. For the $2s$ level it is:

$$E=2\pi^2/r_0^2, \qquad (58)$$

t node of (56) lies at $r_0\sqrt{2E}=4.4934$

$$E=10.0953/r_0^2 \qquad (59)$$

level. All three asymptotes are indicated
s. The $r_0=0$ axis is evidently also an
urves calculated.

a function of r_0, the asymptotes (57),
ght lines through the origin, being there
ing $(E^{-\frac{1}{2}}, r_0)$ curves. It is easy to find
the (E, r_0) curve.

n tables II–IV with the indication § 3*f*
3.

rom section *c* of this paragraph that the
$\rightarrow 0$ are the same as for a spherical box,
le that the quantum mechanical average
, as with the box.

that $\lim_{r_0\rightarrow 0}\overline{V}/E=0$. Taking the value of

$\lim_{r_0\rightarrow 0}\overline{V}$ for the $1s$ level as an example we have by (56):

$$\overline{V}=-\int_0^{r_0}J_{\frac{1}{2}}^2\,(\pi r/r_0)r^{-1}r^2dr\Big/\int_0^{r_0}J_{\frac{1}{2}}\,(\pi r/r_0)r^2dr. \qquad (60)$$

Integration gives:

$$\overline{V}=-\{C+\ln\pi-Ci(2\pi)\}r_0^{-1}=-2.4422r_0^{-1}. \qquad (61)$$

with $C=0.5577$, the constant of Euler–Mascheroni, and $Ci(2\pi)$ the cosine integral for the argument 2π equalling -0.0271.

For $r_0\rightarrow 0$ the potential energy $\overline{V}\rightarrow-\infty$, but because E is proportional to r_0^{-2} it is really so that $\lim\overline{V}/E=0$.

h) The concordance between the set of levels at $r_0\rightarrow 0$ and at $r_0=\infty$ is very simple. The n-fold degeneracy of the levels at $r_0=\infty$, where $N=n$, is removed when r_0 is finite, until at $r_0\rightarrow 0$ the wave function is asymptotically $J_{l+\frac{1}{2}}$ with $l=0,1,\ldots.\,n-1$ and with $N-l-1$ nodes between its limiting points. On the other hand the wave functions $J_{1/2}$ correspond to the $1s$, $2s$, $3s$, etc. levels with increasing number of zero-points, whereas $J_{3/2}$ belongs to the p levels etc.

§ 4. *Possible physical importance of the problem.* Michels, De Boer and Bijl have investigated the ground level of the encaged hydrogen atom for fairly large values of r_0 in order to get an idea of the influence of pressure on the wave function and by consequence on the polarizibility. The study of higher levels might give some information about the shift of spectral lines under pressure. It must however be taken in mind that the procedure followed is a very rough one. In the first place replacing of the influence of pressure by the action of an infinitely high and steep potential wall neglects the effect of Van der Waals attraction forces between molecules, and gives only an idea of the effect of repulsion forces, that act at very high density. Also the resonance broadening of levels when atoms of the same kind come close together is not taken into account, so that we must think that the cage around the hydrogen atom considered, does not consist of hydrogen atoms.

It may however all the same be concluded that the shift of spectral wave lengths can be a considerable one under a pression of some hundreds or thousands of atmospheres. This is visible in figure 3, because $r_0=7$ corresponds with 111 atmospheres and $r_0=5$ with